Color Constancy

- bottom-up color constancy
- top-down color constancy
- color constant features
Color Constancy Research in Human Vision

Often Mondrian images were used as stimuli in color constancy experiments. Humans were asked to match patches in the scene to isolated patches under white light.

From these images the importance of color statistics, spatial mean, maximum flux for color constancy was established.

Human color constancy was still only partially explained by these experiments.

Drawbacks: do not resemble real 3D surfaces, no interreflections, no specularities, shading etc.

Edwin Lan. The retinex, Am Sci 1964
Anya Hurlbert: Is colour constancy real ? Current Biology 1999
Kraft and Brainard designed a more realistic setting for color constancy. Where illuminant and test patch color could be adjusted.

Observers task to adjust the colour of the test patch to be achromatic.

Successive subtraction of cues found them all to be important
- local contrast
- global contrast
- interreflections, specularities

Anya Hurlbert: Is colour constancy real? Current Biology 1999
Color Constancy Research in Human Vision

Observers were asked to adjust the colors of fruits to make them achromatic.

Fruits were considered grey when they physically had a color opposite to its natural color.

Color Constancy at a Pixel
problem statement

How do we recognize colors to be the same under varying light sources?

color constancy: the ability to recognize colors of objects invariant of the color of the light source.
Colour constancy algorithms

Invariant Normalizations
color constancy at a pixel

Assumptions:

1. Lambertian model:
   - linear relation pixel values and intensity light.
   - no specularities and interreflections.
2. perfectly narrow-band sensors (Dirac delta functions).
3. the illuminants are Planckian.

However, the final algorithm is shown to be robust to deviations from the assumptions.
Surface reflectance

\[ e(\lambda) \]

\[ s(\lambda) \]

\[ c_b(\lambda) \]
Dirac delta functions

\[ p_k = \int_{\omega} e(\lambda) c_b(\lambda) s_k(\lambda) d\lambda \]

assumption: Dirac sensors

\[ p_k = \int_{\omega} e(\lambda) c_b(\lambda) q_k \partial(\lambda - \lambda_k) d\lambda \]

\[ p_k = e(\lambda_k) c_b(\lambda_k) q_k \]
Planckian illuminants

**Planck's law** of black body radiation states the spectral intensity of electromagnetic radiation from a black body at temperature $T$ as a function of wavelength:

$$ E(\lambda, T) = \frac{c_1}{\lambda^5} e^{\frac{c_2}{T\lambda}} $$

**Wien’s approx:**

The **Planckian locus** is the path that the color of a black body as the blackbody temperature changes.
Planckian illuminants

**Planck's law** of black body radiation states the spectral intensity of electromagnetic radiation from a black body at temperature $T$ as a function of wavelength:

\[
E(\lambda, T) = \frac{c_1}{\lambda^5} e^{\frac{c_2}{T\lambda}}
\]

Wien's approx: 

The **Planckian locus** is the path that the color of a black body as the blackbody temperature changes.

**Daylight illuminants** can be approximated by Planckian illuminants.

( indoor illuminants to some extend

2500K Household light bulbs
3000K Studio lights, photo floods
4000K Clear flashbulbs
5000K Typical daylight; electronic flash )
Color constancy at a pixel

Planckian light

\[ p_k = e(\lambda_k)c_b(\lambda_k)q_k \quad \rightarrow \quad p_k = \frac{c_1}{\lambda_k^5} e^{\frac{c_2}{T\lambda}} c_b(\lambda_k)q_k \]

Consider the logarithm of the chromaticity coordinates:

\[ \chi_j = \log\left(\frac{p_k}{p_p}\right) = \log\left(\frac{\lambda^{-5} e^{\frac{c_2}{T\lambda}} c_b(\lambda_k)q_k}{\lambda^{-5} e^{\frac{c_2}{T\lambda}} c_b(\lambda_p)q_p}\right) \]

\[ \chi = \text{s} + \frac{1}{T} \text{e} \quad \leftrightarrow \quad \chi_j = \log\left(\frac{s_k}{s_p}\right) + \frac{1}{T} \left( e_k - e_p \right) \]

- \( \chi \) depends on surface color
- \( \chi_j \) depends on illuminant color

\[ s_k = \lambda_k^{-5} c_b(\lambda_k)q_k \]

\[ e_k \equiv -\frac{c_2}{\lambda_k} \]
color constancy at a pixel - examples

examples log chromaticity plots:

Macbeth Color Checker  HP912 Digital Still Camera  Nikon D-100

illuminant invariant direction axis

illuminant variant axis (camera dependent)

Every pixel can be represented in a illuminant invariant representation!
Since shadows are a change in illuminant these representation are shadow free.
shadow detection

Comparison of the edge maps of the original and the shadow invariant image allows for shadow detection.
examples:

- sky and sun light
- sky light
- removal of colored shadow
- shading is not effected
references:

Gamut Mapping
“In real-world images, for a given illuminant, one observes only a limited number of different colors.”

Solux 4700K  
Solux 4700K + Roscolux filter  
Sylvania Warm White Fluorescent

Slide credit: Theo Gevers
Gamut mapping algorithm:

- Obtain input image.
Gamut mapping algorithm:

- Obtain input image.
- Compute gamut from image.

Slide credit: Theo Gevers
Gamut mapping algorithm:

• Obtain input image.
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Gamut mapping algorithm:

- Obtain input image.
- Compute gamut from image.
- Determine feasible set of mappings from input gamut to canonical gamut.
- Apply some estimator, to select one mapping from this set.
- Use mapping on input image to recover the corrected image, or on canonical illuminant to estimate the color of the unknown illuminant.

Slide credit: Theo Gevers
Color Constancy from Color Derivatives
Color Constancy

color constancy: the ability to recognize colors of objects invariant of the color of the light source.

Grey world hypothesis: the average reflectance in a scene is grey.

White patch hypothesis: the highest value in the image is white.

Grey-world: \[ \sum_{m=1}^{M} f_i(x) \propto c \]

white-patch: \[ \left( \sum_{m=1}^{M} f_i(x) \right)^{\frac{1}{\infty}} \propto c \]

Shades of Grey hypothesis: the n-Minkowsky norm based average of a scene is achromatic.

- unifies Grey-World and White Patch: \[ e^p \approx \frac{p}{\sqrt{\int |f(x)|^pdx}} \]
Color Constancy
Color Constancy
Color Constancy
Color Constancy

color constancy: the ability to recognize colors of objects invariant of the color of the light source.

Grey world hypothesis: the average reflectance in a scene is grey.

White patch hypothesis: the highest value in the image is white.

generalization I: the L-norm:
\[ \left( \sum_{m=1}^{M} \left( f_i(x) \right)^k \right)^{\frac{1}{k}} \propto c \]

Grey edge hypothesis: the average edge in a scene is grey.

generalization II: L-norm + differentiation order:
\[ \left( \sum_{i=1}^{M} \left| \frac{\partial^n f_i(x)}{\partial x^n} \right|^p \right)^{\frac{1}{p}} \propto c \]
Color Constancy in 4 lines of matlab code!

Function Illuminant=GreyEdgeCC(im,mink,sigma,dif)

im  =  gauss_derivative(im,sigma,dif);
im  =  reshape(im,size(im,1)*size(im,2),3);
Illuminant= 1./power( sum ( power( im, mink ) ), 1/mink );
Illuminant = Illuminant./norm(Illuminant) ;
Low-level color constancy:

\[ \left( \sum_{i=1}^{M} \left| \frac{\partial^{n} f_i(x)}{\partial x^n} \right|^{p} \right)^{\frac{1}{p}} \propto c \]

- \( n = 0, p = 1 \)
  - \( \sum_{m=1}^{M} f_i(x) \propto c \)
  - grey-world

- \( n = 0, p = \infty \)
  - \( \left( \sum_{m=1}^{M} (f_i(x))^\infty \right)^{\frac{1}{\infty}} \propto c \)
  - white-patch

- \( n = 0, p = k \)
  - \( \left( \sum_{m=1}^{M} (f_i(x))^k \right)^{\frac{1}{k}} \propto c \)
  - shades-of-gray

- \( n = 1, p = 1 \)
  - \( \left( \sum_{m=1}^{M} \left| \frac{\partial f_i(x)}{\partial x} \right|^{p} \right)^{\frac{1}{p}} \propto c \)
  - grey-edge

G. Finlayson, E. Trezzi, “Shades of gray and colour constancy”, CIC 2004
Color Constancy: experiment

- test set: 23 objects under 11 illuminants (Computational Vision Lab: Simon Fraser)

- angular error = $\cos(\hat{e} \cdot e)$
Color Constancy: experiment

<table>
<thead>
<tr>
<th>Method</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grey-World</td>
<td>9.8</td>
</tr>
<tr>
<td>White-Patch</td>
<td>9.2</td>
</tr>
<tr>
<td>General Grey-World</td>
<td>5.4</td>
</tr>
<tr>
<td>Grey-Edge</td>
<td>5.6</td>
</tr>
<tr>
<td>2nd order Grey-Edge</td>
<td>5.2</td>
</tr>
<tr>
<td>Color by Correlation</td>
<td>9.9</td>
</tr>
<tr>
<td>Gamut Mapping</td>
<td>5.6</td>
</tr>
<tr>
<td>GCIE, 11 Lights</td>
<td>4.9</td>
</tr>
<tr>
<td>GCIE, 87 Lights</td>
<td>5.3</td>
</tr>
</tbody>
</table>
Color Constancy: experiment

- real-world data set (F. Ciurea and B. Funt : Vision Lab - Simon Fraser)
Color Constancy: experiment

- real-world data set (F. Ciurea and B. Funt: Vision Lab - Simon Fraser)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grey-World</td>
<td>7.3</td>
</tr>
<tr>
<td>White-Patch</td>
<td>6.7</td>
</tr>
<tr>
<td>General Grey-World</td>
<td>4.7</td>
</tr>
<tr>
<td>Grey-Edge</td>
<td>4.1</td>
</tr>
<tr>
<td>2nd order Grey-Edge</td>
<td>4.3</td>
</tr>
</tbody>
</table>
“In real-world images, for a given illuminant, one observes only a limited number of different colored edges.”

Experiments (real-world images)

Some examples:

<table>
<thead>
<tr>
<th>Original</th>
<th>Ideal</th>
<th>Derivative-based</th>
<th>Regular Gamut</th>
</tr>
</thead>
</table>

How do you choose the best cc-algorithm?
High-Level Color Constancy
Could it be that different scenes prefer different color constancy methods?

Geusebroek and Smeulders (2005) – Weibulls

Examples:
Natural Image Statistics

Distribution of edge responses follows Weibull distribution.

Two parameters:

- $\beta$ – Contrast of the image. A higher value indicates more contrast.
- $\gamma$ – Grain size. A higher value indicates more fine textures.

slide credit: Arjan Gijsenij
Postsupervised Prototype Classification:
Compute Weibull-parameters for all images
Color Constancy – Selection

Postsupervised Prototype Classification:
Compute Weibull-parameters for all images
Partition weibull-parameters using $k$-means

slide credit: Arjan Gijsenij
Color Constancy – Selection

Postsupervised Prototype Classification:

Compute Weibull-parameters for all images

Partition weibull-parameters using $k$-means

Label cluster centers according to the minimum mean angular error

slide credit: Arjan Gijsenij
Color Constancy – Selection

Postsupervised Prototype Classification:
Compute Weibull-parameters for all images
Partition weibull-parameters using $k$-means
Label cluster centers according to the minimum mean angular error
Build 1-NN Classifier on these cluster centers

slide credit: Arjan Gijsenij
Experiments

Data set consisting of 11000+ images

The *true* illuminants are known (ground truth)

Grey sphere is *masked during experiments*

Performance measure → **angular error:**

$$\cos^{-1}(\hat{e}_l \cdot \hat{e}_e)$$
# Experiments – Results

<table>
<thead>
<tr>
<th>Original</th>
<th>Ideal</th>
<th>Selection</th>
<th>White-Patch</th>
<th>Grey-World</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image 1" /></td>
<td><img src="image2.png" alt="Image 2" /></td>
<td><img src="image3.png" alt="Image 3" /></td>
<td><img src="image4.png" alt="Image 4" /></td>
<td><img src="image5.png" alt="Image 5" /></td>
</tr>
<tr>
<td><img src="image6.png" alt="Image 6" /></td>
<td><img src="image7.png" alt="Image 7" /></td>
<td><img src="image8.png" alt="Image 8" /></td>
<td><img src="image9.png" alt="Image 9" /></td>
<td><img src="image10.png" alt="Image 10" /></td>
</tr>
<tr>
<td><img src="image11.png" alt="Image 11" /></td>
<td><img src="image12.png" alt="Image 12" /></td>
<td><img src="image13.png" alt="Image 13" /></td>
<td><img src="image14.png" alt="Image 14" /></td>
<td><img src="image15.png" alt="Image 15" /></td>
</tr>
</tbody>
</table>

*slide credit: Arjan Gijsenij*
## Experiments – Performance

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grey-World</td>
<td>7.9°</td>
<td>7.0°</td>
</tr>
<tr>
<td>White-Patch</td>
<td>6.8°</td>
<td>5.3°</td>
</tr>
<tr>
<td>General Grey-World</td>
<td>6.2°</td>
<td>5.3°</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;-Order Grey-Edge</td>
<td>6.2°</td>
<td>5.2°</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;-Order Grey-Edge</td>
<td>6.1°</td>
<td>5.2°</td>
</tr>
<tr>
<td>Gamut mapping</td>
<td>8.5°</td>
<td>6.8°</td>
</tr>
<tr>
<td>Color-by-Correlation</td>
<td>6.4°</td>
<td>5.2°</td>
</tr>
</tbody>
</table>

slide credit: Arjan Gijsenij
## Experiments – Performance

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd-Order Grey-Edge (baseline)</td>
<td>6.1°</td>
<td>5.2°</td>
</tr>
<tr>
<td>Selection – 5 methods</td>
<td>5.7° (-7%)</td>
<td>4.7° (-10%)</td>
</tr>
<tr>
<td>Combining – 5 methods</td>
<td>5.6° (-8%)</td>
<td>4.6° (-12%)</td>
</tr>
<tr>
<td>Combining – 75 methods</td>
<td>5.0° (-18%)</td>
<td>3.7° (-29%)</td>
</tr>
</tbody>
</table>

*slide credit: Arjan Gijsenij*
Color Constancy from High-Level Visual Information
problem statement

How do we recognize colors to be the same under varying light sources?

color constancy: the ability to recognize colors of objects invariant of the color of the light source.
computational color constancy

- White-Patch
  - Land, 1976

- Grey-World
  - Buchsbaum, 1980

- Gamut Mapping
  - Forsyth, 1990

- Color-by-Correlation
  - Finlayson, 2001

bottom-up approaches!
top-down color constancy

psychophysical motivation:

problem statement

How do we recognize colors to be the same under varying light sources?

color constancy: the ability to recognize colors of objects invariant of the color of the light source.

How can we apply high-level visual information for computational color constancy?
overview our approach

compute semantic likelihood for all images, and select most likely.

output image
**plsa-based image segmentation**

- We use Probabilistic Latent Semantic Analysis (pLSA) to compute the semantic likelihood of an image.

**Image representation**
- dense extraction of 20x20 pixel patches on 10x10 pixel grid
- each patch described by discretized features, the words .
  - texture: SIFT (750 visual words, k-means)
  - color: hue (100 visual words, k-means)
  - position: patch location indicated by cell in a 8x8 grid
We use Probabilistic Latent Semantic Analysis (pLSA) to compute the semantic likelihood of an image.

An image is modeled as a mixture of semantic topics:

\[ p(w|d) = \sum_z p(w|z) p(z|d) \]

- visual word
- image
- semantic topics

The \( p(w^m|z) \) can either be learned supervised or unsupervised. We assume them to be learned from images taken under a white illuminant.

likelihood image

\[ p(d) = \prod_w p(w|d) \]
plsa-based image segmentation

supervised learning

unknown

test image

semantic image segmentation

\[ p(w|d) = \sum_z p(w|z)p(z|d) \]

using EM: \( p(z|d) = \{0.6, 0.4\} \)
plsa-based image segmentation

unsupervised learning

\[ p(w|d) = \sum_z p(w|z) p(z|d) \]

\[ p(w|c1) \]

\[ p(w|c2) \]

\[ p(w|z) \]

unknown

using EM: \[ p(z|d) = \{0.6, 0.4\} \]

semantic image segmentation
semantic likelihood image

E = -14.1

bike

sky

grass

plane

E = -13.5

sky

grass

plane

E = -14.5

water

plane

grass

plsa-analysis

color constancy hypothesis
casting hypotheses: bottom-up

Low-level color constancy:

\[ \left( \sum_{i=1}^{M} \left| \frac{\partial^n f_i(x)}{\partial x^n} \right|^p \right)^{\frac{1}{p}} \propto c \]

We will use \( n=\{0, 1, 2\} \) and \( p=\{2, 12\} \) to cast a total of 6 bottom-up hypotheses.

- \( n = 0, p = 1 \):
  \[ \sum_{m=1}^{M} f_i(x) \propto c \]
  grey-world

- \( n = 0, p = \infty \):
  \[ \left( \sum_{m=1}^{M} \left| \frac{\partial f_i(x)}{\partial x} \right| \right)^{\frac{1}{p}} \propto c \]
  white-patch

- \( n = 0, p = k \):
  \[ \left( \sum_{m=1}^{M} \left| \frac{\partial f_i(x)}{\partial x} \right|^k \right)^{\frac{1}{k}} \propto c \]
  shades-of-gray

- \( n = 1, p = 1 \):
  \[ \left( \sum_{i=1}^{M} \left| \frac{\partial f_i(x)}{\partial x} \right| \right)^{\frac{1}{p}} \propto c \]
  grey-edge

G. Finlayson, E. Trezzi, “Shades of gray and colour constancy”, CIC 2004
J. van de Weijer, T. Gevers “Edge-Based Color Constancy”, IEEE TIP 2007
casting hypotheses: top-down

apply PLSA based on texture and position to assign pixels to classes

cast one illuminant hypothesis for each detected class

compute semantic likelihood for all hypotheses, and select most likely

green grass hypothesis: the average reflectance of a semantic class in an image is equal to the average of the semantic class in the train-set
Data Set contains both indoor and outdoor scenes from a wide variety of locations (150 training, 150 testing)

Topic-word distributions are learned unsupervised on the texture and position cue (color is ignored in training).

experiment: illuminant estimation

results in angular error:

<table>
<thead>
<tr>
<th></th>
<th>standard color constancy</th>
<th>high-level selection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no cc</td>
<td>worst BU</td>
</tr>
<tr>
<td>indoor</td>
<td>12.8</td>
<td>12.3</td>
</tr>
<tr>
<td>outdoor</td>
<td>5.5</td>
<td>7.4</td>
</tr>
</tbody>
</table>

input image | bottom-up | top-down

1.8          | 7.8       | 1.4
22.1         | 4.5       | 0.5
experiment: semantic segmentation

Data Set training: labelled images of Microsoft Research Cambridge (MSRC) set, together with ten images collected from Google Image for each class. Training: 350 images. Test: 36 images.

Topic-word distributions are learned supervised.

Classes: building, grass, tree, cow, sheep, sky, water, face and road.

experiment: pixel classification

results pixel classification in %:

<table>
<thead>
<tr>
<th></th>
<th>standard color constancy</th>
<th>high-level selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>no cc</td>
<td>worst BU</td>
<td>best BU</td>
</tr>
<tr>
<td>39.6</td>
<td>41.4</td>
<td>52.2</td>
</tr>
<tr>
<td></td>
<td>BU</td>
<td>TD</td>
</tr>
<tr>
<td></td>
<td>53.4</td>
<td>59.5</td>
</tr>
</tbody>
</table>

![Images of results with labels: tree, grass, face, grass.]
Blur Robust and Color Constant

Image description
problem statement

How do we recognize colors to be the same under varying light sources?

color constancy: the ability to recognize colors of objects invariant of the color of the light source.

Change of illuminant can be modeled by the *diagonal model*. 
Colour constancy algorithms

Invariant Normalizations

Illuminant estimation

Colour rectification

Normalization

Normalization

Normalization

Normalization

slide credit: R. Baldrich
Color Constant Derivatives

- A color constant representation of a single color patch is impossible.
- The difference between two color patches can be represented invariant to the color illuminant.

Funt and Finlayson:

Mondrian-world: \( f(x) = m^b c^b (x) e \)

\[
p = \frac{R^1}{R^2} = \frac{m^b c_1^R e^R}{m^b c_2^R e^R} = \frac{c_1^R}{c_2^R}
\]

\[
\ln p = \ln \frac{R^1}{R^2} = \ln R^1 - \ln R^2 = \frac{\partial}{\partial x} \ln R
\]

Gevers and Smeulders:

3D-world: \( f(x) = m^b (x) c^b (x) e \)

\[
m = \frac{R^1 G^2}{R^2 G^1} = \frac{m_1^b c_1^R e^R}{m_2^b c_2^R e^R} \frac{m_2^b c_2^G e^G}{m_1^b c_1^G e^G} = \frac{c_1^R c_2^G}{c_2^R c_1^G}
\]

\[
\ln m = \ln \frac{R^1 G^2}{R^2 G^1} = \ln \frac{R^1}{G^1} - \ln \frac{R^2}{G^2} = \frac{\partial}{\partial x} \ln \frac{R}{G}
\]
Color Constant Derivatives

- A color constant representation of a single color patch is impossible.
- The difference between two color patches can be represented invariant to the color illuminant.

These theories overlook the fact that an edge operator measures two properties of the edge:

1. the color difference
2. the steepness of the edge

\[
p = \frac{R}{R^2} = \frac{m^b c_2 e^R}{m^b c_2 e^R} = \frac{c_1}{c_2}
\]

\[
\ln p = \ln \frac{R^1}{R^2} = \ln R^1 - \ln R^2 = \frac{\partial}{\partial x} \ln R
\]

\[
m = \frac{RG^1}{R^2 G^1} = \frac{m^b c_2 e^R}{m^b c_2 e^R} = \frac{c_1}{c_2}
\]

\[
\ln m = \ln \frac{R^1 G^2}{R^2 G^1} = \ln \frac{R^1}{G^1} - \ln \frac{R^2}{G^2} = \frac{\partial}{\partial x} \ln \frac{R}{G}
\]
Why is this a problem?

- Image blur is frequently encountered phenomenon.
- Possible causes are: out-of-focus, relative motion between camera and object, and aberrations of the optical system.
Obtaining Invariance to Image Blur

- A color constant representation of a single color patch is impossible.
- The difference between two color patches can be represented invariant to the color illuminant.

Funt and Finlayson:

Mondrian-world: \( f(x) = m^b c^b e(x) \)

\[
p = \frac{R^1}{R^2} = \frac{m^b c^1 R e^R}{m^b c^2 R e^R} = c^R_1 \]

\[
\ln p = \ln \frac{R^1}{R^2} = \ln R^1 - \ln R^2 = \frac{\partial}{\partial x} \ln R
\]

Consider a blurred image: \( R' = R \otimes G^\sigma \)

\[
\frac{\partial}{\partial x} \sigma^d \ln R = \frac{R_x^{\sigma^d}}{R^{\sigma^d}} \quad \frac{\partial}{\partial x} \sigma \ln R' = \frac{R_x^{\sqrt{\sigma^2 + \sigma^2}}}{R^{\sqrt{\sigma^2 + \sigma^2}}}
\]

On the edge the following holds:

\[
R^{\sqrt{\sigma^2}} = R^{\sqrt{\sigma^2 + \sigma^2}} \quad R_x^{\sqrt{\sigma^2}} = C(\sigma) R_x^{\sqrt{\sigma^2 + \sigma^2}}
\]

Robustness with respect to blur is obtained by:

\[
\varphi_p^l = \arctan \left( \frac{R_x G}{G_x R} \right) \quad \varphi_p^l = \arctan \left( \frac{G_x B}{B_x G} \right)
\]
Retrieval Experiment I

• Twenty different objects were captured under 11 different object orientations and 11 different light sources (Simon Fraser).
• We compare the retrieval results of the color constant description with the color constant and blur robust description.
• Error given in Normalized Average Rank (NAR).

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<th>rank</th>
<th>ANAR</th>
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<tr>
<td>$\varphi_p$</td>
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<tr>
<td>m</td>
<td>155</td>
</tr>
<tr>
<td>$\varphi_m$</td>
<td>115</td>
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Retrieval Experiment II

- Twenty pairs of images with varying image blur.
- We compare the retrieval results of the color constant description with the color constant and blur robust description.

<table>
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<th>rank</th>
<th>p</th>
<th>ϕₚ</th>
<th>m</th>
<th>ϕₘ</th>
<th>ANAR</th>
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<td>2</td>
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Summary Color Constancy

• The Planckian locus describes natural light illuminants.

• Color constancy at the pixel allows for shadow removal.

• The general grey-world algorithm generalizes a set of low-level color constancy algorithms, including white patch, grey-world, grey-edge, and shades —of-grey.

\[
\left( \sum_{i=1}^{M} \left| \frac{\partial^n f_i(x)}{\partial x^n} \right|^p \right)^{\frac{1}{p}} \propto c
\]

• Top-down information improves both color constancy performance and semantic segmentation results.
references: color constancy